Localized bases for kernel spaces: recent progress

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A well known challenge for kernel approximation is the conditioning of the standard kernel basis: as the dimensions of the underlying spaces increase, the practicality of working directly with linear combinations of kernels decreases. Localized bases, an idea born from earlier preconditioners developed by Powell and Beatson [1], can effectively treat this basic challenge by replacing the standard basis with a rapidly decaying, easily constructed, stable basis of approximate Lagrange functions. The success of this method at large scales comes from the stationary decay of Lagrange functions: analytic observations made in [2, 3].

In recent years we have shown that different kernels in diverse settings admit localized bases (Sobolev kernels on compact Riemannian manifolds, restricted thin plate splines on spheres, surface splines and Matérn kernels on R^d). In this talk, I will present recent results in this topic, including improved approximation rates, inverse estimates, applications to PDEs, and work on non-quasi-uniform arrangements of centers.

Joint work with: Fran Narcowich, Christian Rieger, Joseph Ward.

References

[1] R. K. Beatson and M. J. D. Powell, *An iterative method for thin plate spline interpolation that employs approximations to Lagrange functions*, in Numerical analysis 1993 (Dundee, 1993), vol. 303 of Pitman Res. Notes Math. Ser., Longman Sci. Tech., Harlow, 1994, pp. 17–39.

[2] T. Hangelbroek, F. J. Narcowich, and J. D. Ward, *Kernel approximation on manifolds I: Bounding the Lebesgue constant*, SIAM Journal on Mathematical Analysis, 42 (2010), pp. 1732–1760.

[3] T. Hangelbroek, F. J. Narcowich, X. Sun, and J. D. Ward, *Kernel approximation on manifolds II: The L_∞ norm of the L_2 projector*, SIAM Journal on Mathematical Analysis, 43 (2011), pp. 662–684.