

# B-spline-like simplex spline bases on the Powell-Sabin 12-split

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For spaces of constant, linear, and quadratic splines of maximal smoothness on the Powell-Sabin 12-split of a triangle, Cohen, Lyche and Riesenfeld recently discovered so-called S-bases [1]. Later, S-bases of higher smoothness [2] and on the Clough-Tocher split [3] were discovered. These S-bases are simplex spline bases with B-spline-like properties on a single macrotriangle, which are tied together across macrotriangles in a Bézier-like manner.

In this presentation we extend these results to the space of  $C^2$  cubic splines on the 12-split, as described in [4]. We give a formal definition of an S-basis in terms of certain basic properties. We proceed to investigate the existence of S-bases for the aforementioned spaces on the 12-split and additionally the cubic case, resulting in an exhaustive list. From their nature as simplex splines, we derive simple differentiation and recurrence formulas to other S-bases. We establish a Marsden identity that gives rise to various quasi-interpolants and domain points forming an intuitive control net, in terms of which conditions for  $C^0$ ,  $C^1$ , and  $C^2$ -smoothness are derived.

Although the cubic bases on the 12-split can only be used to define smooth surfaces over specific triangulations, we envision applications for local constructions, such as hybrid meshes and extra-ordinary points.

**Joint work with:** Tom Lyche.

## References

- [1] Elaine Cohen, Tom Lyche, Richard F. Riesenfeld. A B-spline-like basis for the Powell-Sabin 12-split based on simplex splines. *Math. Comp.*, 82, no. 283: 1667–1707, 2013.
- [2] Tom Lyche and Georg Muntingh. Stable simplex spline bases for  $C^3$  quintics on the Powell-Sabin 12-split. *Constr. Approx.*, February 2017, Volume 45, Issue 1: 1–32, 2015.
- [3] Tom Lyche and Jean-Louis Merrien. Simplex-Splines on the Clough-Tocher Element. Preprint available at <https://hal.archives-ouvertes.fr/hal-01677442>
- [4] Tom Lyche and Georg Muntingh. B-spline-like bases for  $C^2$  cubics on the Powell-Sabin 12-split. Preprint, 2018.