Sparse G^1 Spline Manifold and Salient Feature Map Approximations with Topological Accuracy

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We approximate a surface manifold, (or a point set sampling of a surface manifold), and recover salient features of the manifold by a sparse set of G^1 B-spline functions. The final solution possesses the same topology as that of the original manifold as well as the recovered salient features while providing a sparse representation using smooth spline functions supported on quad meshes. We utilize an alternating optimization method, between progressively improving a quad mesh with well distributed sampling of vertices using ϵ-nets (or *Delone* sets) [AHPV05, Cla06, LL00] and approximating the surface manifold (or point set) based on the use of compactly supported G^1 spline function basis. For the first alternating step we optimize the coarse complex approximation and infer the topology of the manifold using a sparsified Rips complex, along with the homology recovery of salient shape features [CdS03, She, DFY15]. The second alternating optimization step produces a sparse parametric encoding of the surface manifold and of recovered salient features. We use distribution sampling strategies for producing low discrepancy sampling of vertices and also allow for a choice of different atomic G^1 spline function basis [BXHN02, BMV17, TZ11, YZ04, WLY$^+$16].

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