Poster: Laplace–Beltrami Operator on Digital Surfaces

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Our model of surface comes from the Digital Geometry theory [5], where the discrete structure is the topological boundary of a subset of point in $\mathbb{Z}^d$ called a *digital surface* (and example of this object is pictured in Fig. 1). Such surfaces can be constructed from mathematical modeling or from boundaries of partitions in volumetric images. Digital objects naturally arise in many material sciences of medical imaging applications as tomographic volumetric acquisition devices usually generate regularly spaced data.

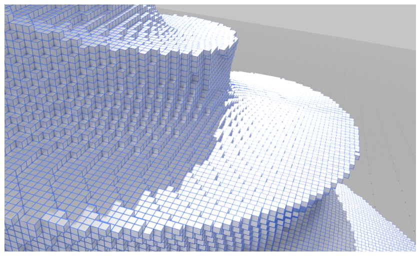

Figure 1: A digital surface of dimension two embedded in $\mathbb{R}^3$.

Our goal here is to present a discretization of the Laplace–Beltrami operator on digital surfaces which satisfies strong consistency (*i.e.* pointwise convergence) with respect to the Laplace–Beltrami operator on the underlying manifold when the digital surface is the boundary of the digitization of a continuous object. Existing techniques (such as the cotan Laplace) yield poor results on our datas: the usual assumption is that the geometry of the surface captures in some way the underlying metric of the smooth surface. This is not the case for digital surfaces: for example the sum of the area of quads does not converge to the real area of the underlying manifold (which is a key element in the discretization of the operator).

In response, we adapt the operator of Belkin *et al.* [1] to our specific data. The method uses an accurate estimation of areas associated with digital surface elements. This estimation is achieved through the multigrid convergent digital normal estimator of Coeurjolly *et al.* [4]. The proposed poster sums up the work of [2] where we investigated applications such as heat diffusion or shape approximation through the eigenvectors decomposition (see Fig. 2) of the linear operator and the work of [3] where we proved strong consistency, compared our approximation with many others on triangulated surfaces, polygonal surfaces and digital surfaces and illustrated the operator with an application to mean curvature.

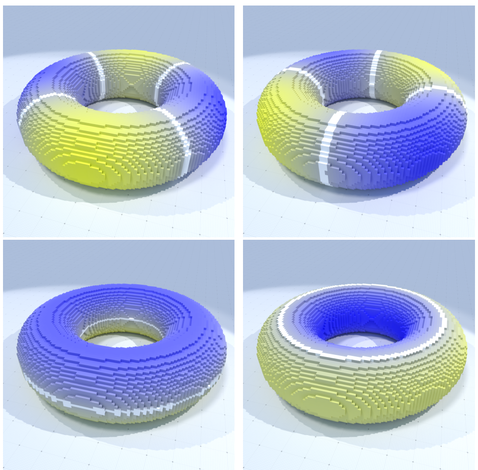

Figure 2: The fourth, fifth, sixth and seventh eigenfunction of the Laplace Operator on a Digital Torus.

Joint work with: David Coeurjolly, Jacques-Olivier Lachaud and Tristan Roussillon.

References

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