Geometrical questions in low-rank optimization

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In low-rank optimization one seeks to minimize functions on manifolds or varieties of low-rank matrices or tensors. This has useful applications in signal and image processing, or in high-dimensional scientific computing. The simplest task is the approximation of a tensor by another one of low-rank. Usually, nonlinear optimization methods need to be employed. Due to their parametrization by multilinear maps, sets of low-rank matrices and tensors exhibit a rich geometric structure, which sometimes makes it possible to go beyond generic results in the analysis of optimization methods on these sets. This concerns for example necessary first-order optimality conditions or classification of critical points. If the low-rank optimization problem stems from the discretization of an initially infinite-dimensional problem on a tensor product space (like the discretization of a PDE), another set of questions arises regarding the correct and well-posed variational formulation on the corresponding infinite-dimensional set of low-rank tensors. Topological properties like weak closedness of these sets play a crucial role here. In this talk we will discuss some of these interesting topics.