

# A case of exact recovery using OMP with continuous dictionaries

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We present new theoretical results on sparse recovery guarantees for a greedy algorithm, orthogonal matching pursuit (OMP), in the context of continuous dictionaries. Consider a sparse linear combination of atoms from a dictionary parameterized by some real parameters, for example, a combination of shifted versions of a basic waveform as in the context of sparse spike deconvolution. Currently, performance guarantees for greedy algorithms are typically carried out in the discrete setting associated to a grid of atom parameters, and based on, e.g., the coherence of the considered discretized dictionary [1]. However, such analyses fail to be conclusive for grid-based approaches when the discretization step tends to zero, as the coherence goes to one. Instead, our analysis is directly conducted in the continuous setting. For atoms parametrized by a real parameter that are elements of the (infinite-dimensional) Hilbert space  $L_2(\mathbb{R})$  of square integrable real functions, and such that the inner product between two atoms is the exponential of the negative absolute difference of the corresponding parameters, we show in the noise-free setting that OMP exactly recovers the atom parameters as well as their amplitudes, regardless of the number of distinct atoms. We exhibit a convolutive dictionary of exponentially decaying pulses for which the atoms have an analytic definition while their pairwise inner products have the prescribed form. The established guarantees rely on a proof technique which is the continuous equivalent of Tropp's Exact Recovery Condition (ERC) [1]. The proof exploits specific properties of the positive definite kernel between atom parameters defined by the inner product between the corresponding atoms. Future work will aim at characterizing the class of kernels for which such an analysis holds in particular for higher dimensional parameters and the compatibility of the guarantees with dimension reduction techniques such as sketching, which would pave the way to provably good greedy algorithms for compressive statistical learning [2]. In light of the existing links between Tropp's ERC and recovery guarantees for  $\ell_1$  minimization [3], an interesting question is whether these guarantees extend to sparse spike recovery with total variation norm minimization [4, 5].

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## References

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