EIM in the frame of least-squares optimal interpolation method

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The empirical interpolation method (EIM [1, 2, 3]) has been introduced to extend the reduced basis technique [4, 5, 6] to nonaffine and nonlinear partial differential equations (PDEs). The essential components of EIM procedure are (i) a good hierarchy of collateral reduced basis approximation spaces $(V_n)_n$ with $\dim(V_n) = n$ and $V_n \subset V_{n+1}$, (ii) a series of well selected interpolation points (also called 'magic point') $\{x_i\}_i$ on a domain $\Omega \in \mathbb{R}^d$, and (iii) an effective a posteriori estimator to quantify the interpolation errors. The interpolation in V_n of an unknown continuous function f is defined as $I_n[f] \in V_n$ and $I_n[f](x_i) = f(x_i),\ i = 1, \cdots, n$. The numerical analysis of this method has been presented in [7].

More generally, if more measurements are available than the dimension of V_n, we consider the problem of reconstructing an approximation of f in the reduced basis space V_n from noiseless (or possibly noisy) samples of f at m points $\{x_i\}_{i=1}^m,\ n \leq m$. Recently, the reconstruction with a weighted least-squares approximation in a given linear space V_n and m independent random samples has been studied in [9, 8]. The results show that stable results and optimal accuracy comparable to that of best approximation in V_n are achieved under the mild condition that m scales linearly with n up to an additional logarithmic factor and the points $\{x_i\}_{i=1}^m$ are randomly chosen with respect to a sampling measure which depends on the space $V_n : k_n(x) := \sum_{i=1}^n |\tilde{q}_i(x)|^2$, where $\{\tilde{q}_i\}$ is an orthonormal basis of V_n. Inspired by [8, 9], we first investigate the EIM magic points with respect the function $k_n(x)$. The 1D numerical results show that most of the magic points are actually located on the local extremum of function $k_n(x)$, which seems coherent with the random sampling with the density proportional $k_n(x)$ as is proposed in [8]. Also inspired by [9], we propose a least-square framework for the empirical interpolation method with more points than originally required. Our numerical finding is that, choosing the additional points $\{x_{n+1}, \cdots, x_m\}$, as being the $n+1$-th to m-th EIM magic points, allows to reach an accuracy similar to that of best approximation in V_n with a high stability performance. Furthermore, these additional EIM points are better than a choice of a random sampling or other sampling methods.

References

[1] M. Barrault and Y. Maday and N. C. Nguyen and A. T. Patera. https://doi.org/10.1016/j.crma.2004.08.006.

[2] M. A. Grepl and Y. Maday and N. C. Nguyen and A. T. Patera. https://doi.org/10.1051/m2an:2007031.

[3] Y. Maday and N. C. Nguyen and A. T. Patera and S. H. Pau. https://doi.org/10.3934/cpaa.2009.8.383.

[4] Y. Maday and A. T. Patera and G. Turinici. https://doi.org/10.1023/A:1015145924517.

[5] Y. Maday. https://doi.org/10.4171/022-3/60.

[6] G. Rozza and D. B. P. Huynh and A. T. Patera. https://doi.org/10.1007/s11831-008-9019-9.

[7] Y. Maday and O. Mula and G. Turinici. https://doi.org/10.1137/140978843.

[8] A. Cohen and G. Migliorati. https://doi.org/10.5802/smai-jcm.24.

[9] A. Cohen and M. A. Davenport and D. Leviatan https://doi.org/10.1007/s10208-013-9142-3.