On the H^2-Gradient Flow for the Willmore Energy

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Higher order gradient flows have already been discussed from a phenomenologically perspective, e.g., in [1]. For the Willmore energy and other energies depending on curvature, the classical L^2-gradient flow employs the L^2-inner product $\langle u, v\rangle_{L^2} = \int_M \langle u, v\rangle \,\mathrm{d}\,A$ and leads to a parabolic partial differential equation of order four. Simulating this flow requires time step sizes proportional to the spacial mesh size, rendering them very inefficient for minimization on fine meshes. This mesh dependence completely vanishes for the gradient flow with respect to the H^2-inner product $\langle u, v\rangle_{H^2} = \int_M \langle \Delta u, \Delta v\rangle \,\mathrm{d}\,A$. We explain this by showing that the flow in the smooth setting is governed by an *ordinary* differential equation on a suitable space of immersed surfaces (immersions of Sobolev class $W^{2,p}$ with $p > 2$). Although this space cannot be given the structure of a Riemannian manifold, we argue that $\langle u, v\rangle_{H^2}$ resembles considerably more properties of a Riemannian metric than $\langle u, v\rangle_{L^2}$.

References

[1] I. Eckstein, J.-P. Pons, Y. Tong, C.-C. J. Kuo, and M. Desbrun. Generalized surface flows for mesh processing. In *Proceedings of the Fifth Eurographics Symposium on Geometry Processing*, SGP '07, pages 183–192. Eurographics Association, 2007.

[2] H. Schumacher. On H^2-gradient flows for the Willmore energy, 2017. arXiv:1703.06469