Kernel metrics on shapes representation with normal cycles. Application to registration of curves or surfaces.

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The registration between two curves or two surfaces can be tackled down with a variational formulation:

$$\arg\min_{\varphi \in G} E(\varphi) + A\big(\varphi(C), S\big) \tag{1}$$

where G is a group of deformations, E is an energy that enforces the regularity of the optimal deformation, and A is a data-attachment term that measures the residual distance between the deformed shape $\varphi(C)$ and the target shape S.

In the Large Deformation Diffeomorphic Metric Mapping construction, G is generated through integration of the flows of time-varying vector fields. This is an active research field and in the following, we will rely on such construction ([1]).

Defining the data-attachment term A is of importance since it relaxes the hypothesis of exact matching between two shapes. This is closely related to the question of shapes representation. Indeed, we need first a sound mathematical setting where the shapes are represented, and then an explicit metric in order to have computable expression for A. The representation of shapes with *currents* or *varifolds* have been coupled with kernel metrics for this purpose [5, 2]. However, these two models contain only first order information of the shapes. As a consequence, a matching with such data-attachment terms will be insensitive to features associated with high curvature regions.

In this presentation, we propose to use a second order model to define A, with the representation of shapes with normal cycles. The normal cycle of a shape is an object associated with its unit normal bundle, and it contains all the curvature information of the shape. By defining kernel metrics on normal cycles, we are able to have explicit metric on shapes (continuous and discrete within the same framework), which takes into account specific kind of curvature, depending on the kernel.

If such construction can be used on its own, we fit this distance in the registration framework (equation 1) in order to have a matching between shapes which is sensitive to high curvature regions, but also topological singularities (boundaries, branching points, etc.). We compare our results with the ones obtain with currents or varifolds. One can find more details on this work in [3, 4].

Joint work with: Joan Alexis Glaunès, MAP5, Université Paris-Descartes.

References

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