Optimal control via smoothing splines with additional conditions

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This research is devoted to the problem of optimal control (i.e., determining control and state trajectories) for a linear dynamic system over a period of time to minimize an objective functional. In applications including trajectory planning, the cost functional often is used with the merit to push a trajectory through or nearby a set of desired points since we need to specify the position that the system will be in at a sequence of times. In most situations it is not really crucial that we pass through these points exactly, but rather that we go reasonably close to them, while minimizing the cost functional. Such approach is closely related to the idea of smoothing in the spline theory.

Our aim is to apply methods and techniques developed for smoothing splines to solving optimal control problems in some special cases. We deal with the following linear system

$$x' = Mx + \beta u, \quad y = \gamma^\top x,$$

considered with additional boundary ($y(a) = y_a,\ y(b) = y_b$) or initial ($x(a) = \alpha$) conditions, where x is a vector-valued absolutely continuous function defined on $[a, b]$, M is a given matrix, α, β, γ are given vectors of compatible dimensions and y_a, y_b are given real numbers. We consider this system as the curve $z = y(t)$ generator with the goal to find a control law $u \in L_2[a, b]$ which drives the scalar output trajectory close to a sequence of set points at fixed times $\{(t_i, z_i) : i = 1, 2, \ldots, n\}$, where $a < t_1 < t_2 < \ldots < t_n < b$, and minimizes the objective functional with a positive weight ρ:

$$\int_a^b u^2(t)dt + \rho \sum_{i=1}^{n} (y(t_i) - z_i)^2 \to min.$$

In this report we show how the spline-based methods can be adapted to find an analytic form for solutions of such problem in some special cases of matrix M and vector β. The study is closely related to our previous works [1]-[2] and develops applications of splines in the control theory (see, e.g., [3]-[4]) by including techniques based on smoothing splines with additional conditions.

Joint work with: Natalja Budkina.

References

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[4] J. Shen and X. Wang. A Constrained Optimal Control Approach to Smoothing Splines. *Proceedings of IEEE Conference on Decision and Control, Orlando*, 1729–1734, 2011.