Critical length: an alternative approach

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The *critical length* is a crucial notion attached to kernels of linear differential operators L with constant coefficients, which was first introduced in [2]. Such kernels are known to advantageously replace polynomial spaces because, unlike them, they inherently depend on parameters which can be used to interactively modify the solution to classical problems (e.g., interpolation, design, approximation). Nevertheless, to take benefit of these parameters it may be necessary to restrict the length of the interval $[a, b]$ we are working on. For instance, for a given L, if we are interested in Hermite interpolation, $E := \ker L$ must be an *Extended Chebyshev space on* $[a, b]$. This is ensured if and only the length $b - a$ is less than a fixed number $\ell \in]0, +\infty]$. This number ℓ is referred to as the *critical length* of L (or of E). It is well known that $\ell = +\infty$ if and only if the characteristic polynomial of L has only real roots. If we want to use E for design [5], we have to require E to contain the constants and the length $b - a$ to be less than the critical length of the space DE obtained by differentiation, which is less than or equal to the critical length of E.

We can therefore see the importance of determining the critical length of L when the characteristic polynomial of L has at least one non-real root. The classical approach consists in finding the smallest positive zero of a number of Wronskians attached to L [2, 6]. As an example, in the simplest case of cycloidal spaces (i.e., spaces spanned by polynomials of some degree and the two functions cos and sin) the critical lengths were studied in [2, 3] and definitely identified as zeros of Bessel functions in [4].

Unfortunately, this Wronskian approach is generally difficult to carry out in practice, all the more so as the dimension increases. This motivated us to develop an effective numerical procedure instead, which we will present in this talk. This procedure is obtained as a special case of a numerical test built in [1] to determine whether or not a given space produced by connecting different Extended Chebyshev spaces on adjacent intervals via connection matrices can be used for design. Moreover, an advantage of the proposed algorithm is that it simultaneously provides the Bernstein-type bases which we can use for numerical computations. Examples illustrate the efficiency of this alternative approach both for the computation of the critical lengths and for handling parametric curves.

Joint work with: Giulio Casciola, Marie-Laurence Mazure.

References

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