Intrinsically Defined Curves based on Explicit B-spline Curvature Functions and their Extension to 3D

Norimasa Yoshida
Nihon University
norimasa@acm.org

Controlling the curvature variation of a curve segment with a given G^1 or G^2 Hermite condition is not an easy task. In this talk, we introduce intrinsically defined 2D curves based on explicit B-spline curvature functions[1] and consider their extension to 3D curves. In the proposed 2D curve, its curvature is represented in explicit B-spline form and the curvature is integrated twice (numerical integration is required once, though) to generate a curve segment. Since an explicit B-spline curvature function is represented by *control curvatures*, we present a method for G^1 or G^2 Hermite interpolation by moving control curvatures using an optimization such that the curve satisfies the given condition. Our proposed 2D curves have the following characteristics:

- For a given G^1 or G^2 Hermite interpolation, we can generate various curves satisfying the given condition with controlling its curvature variation.
- The proposed curves can match a wide variety of viable G^2 Hermite conditions
- The proposed curves can be generated in real time using a recent notebook computer.
- The curvature comb of a curve can be directly controlled by moving control curvatures.

Fig. 1 shows examples of G^1 Hermite interpolation of proposed 2D curves using cubic B-spline curvature functions. In this example, the first and last control curvatures are moved to satisfy given G^1 Hermite conditions. In Fig. 1(b), the knot spacing is modified to modify the curvature variation.

We also introduce our idea to extend the proposed curves into 3D curves where both curvature and torsion are represented in explicit B-spline form. We present our ideas to generate 3D curves satisfying G^1 or G^2 conditions. We expect that proposed 3D curves have more degrees of freedom satisfying aesthetic constraints (such as monotonically varying curvature) than log-aesthetic space curves[2].

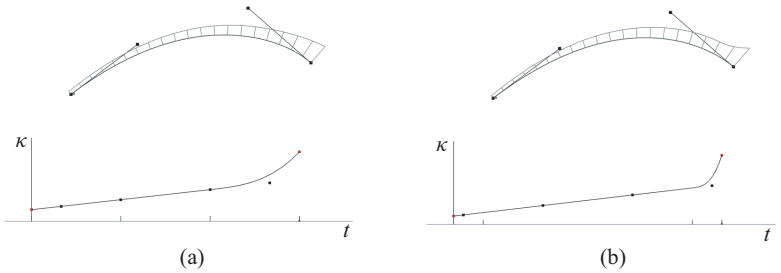

Figure 1: Cubic B-spline curvature function and it's G^1 Hermite interpolation

Joint work with: Takafumi Saito.

References

[1] N. Yoshida, T. Saito. Arc Length Parameterization Curves based on Explicit Polynomial B-splines,. *Conference on Geometry: Theory and Applications*, Chech Republic, Jun. 27, 2017.

[2] N. Yoshida, R. Fukuda, T. Saito. Log-Aesthetic Space Curve Segments, *SIAM/ACM Joint Conference on Geometric and Physical Modeling (GDSPM)*, pp.35-46 2009.