Proposal of
Tangential Angle Paramaterization Space Curves

Tomomi KUDA
Tokyo University of Agriculture and Technology
s144979s@st.go.tuat.ac.jp

Generating aesthetic curve is an important part of industrial and graphical design. Aesthesis of a curve depends on its curvature profile. However, curvature manipulation in existing CAD system is not intuitive.

Tangential Angle Parameterization Curve (TAP curve) is a curve representation method that enables intuitive manipulation of curvature profile [1]. The shape of a TAP Curve is defined from curvature radius. Curvature radius ρ is described as TAP radius function $\rho(\theta)$ where θ is tangential angle. The position of a point on the curve $P(\theta)$ is given by the following equation in the complex plane;

$$P(\theta) = \int_{\theta_0}^{\theta} \rho(\phi) e^{i\phi} d\phi + P_0$$

where P_0 and θ_0 are the position and the tangential angle at the start point. TAP curve using a Bernstein polynomial as TAP radius function is called Bézier-TAP curve. Its curvature profile can be designed by manipulating the control curvature radii of the explicit Bézier (Fig. 1, 2). However, TAP curves generates only planar curves without torsion.

In this talk, we extend the idea of TAP curves to generate space curves. A TAP space curve can be generated by lifting a planar TAP curve along the axis perpendicular to the complex plane (Fig. 3). The lifting height is in proportion to the arc length of the planar TAP curve. A TAP space curve $Q(\theta)$ generated from the TAP curve $P(\theta)$ is given by the following equation;

$$Q(\theta) = \int_{\theta_0}^{\theta} \rho(\phi)(e^{i\phi} + wG_w) d\phi + P_0$$

where w is the unit vector of the lifting direction and G_w is a constant parameter that defines the amount of lifting. The angle between the TAP space curve and the complex plane is $\arctan G_w$ at any point on the curve. On a TAP space curve, the torsion profile is in proportion to the curvature profile.

We have realized G^1 and G^2 Hermite interpolation of Bezier-TAP space curves so that the curve can be generated from the position and the tangent direction at each end point in 3D space.

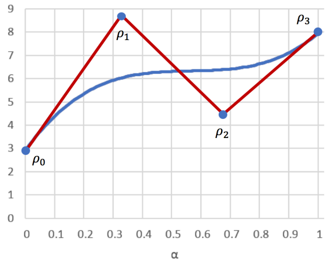

Figure 1: TAP radius function in Explicit Bézier

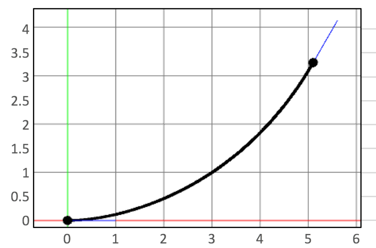

Figure 2: Cubic Bézier-TAP curve

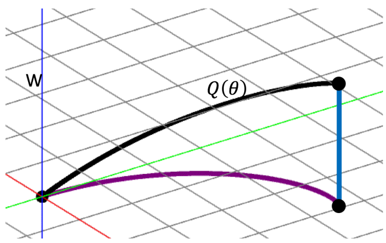

Figure 3: TAP space curve

Joint work with: Takafumi SAITO.

References

[1] T. Saito, N. Yoshida, Proposal of tangential angle parameterizaiton curves, 9th Int'l Conf. on Mathematical Methods for Curves and Surfaces, 2016.