Lifting the Curse of Dimensionality by Quasi-Monte Carlo Methods

Frances Y. Kuo

School of Mathematics and Statistics, University of New South Wales, Sydney, Australia f.kuo@unsw.edu.au

High dimensional problems – meaning problems with hundred or thousands or even infinitely many continuous variables – are now arising in many real world applications. They present major challenges to computational resources, and require serious mathematical efforts in devising new and effective methods. This talk will begin with a contemporary review of Quasi-Monte Carlo (QMC) methods, which offer tailored point constructions for solving high dimensional integration and approximation problems by sampling. By exploiting the smoothness properties of the underlying mathematical functions, QMC methods can achieve higher order convergence rates than standard Monte Carlo sampling, and moreover, QMC error bounds can be independent of the dimension under appropriate theoretical settings.

In recent years the modern QMC theory has been successfully applied to a number of application areas including option pricing in financial mathematics, maximum likelihood estimation in statistics, and PDEs with random coefficients in computational physics and uncertainty quantification. Rather than talking about these well-known popular problems, this talk will present some ongoing works where we take QMC methods to new territories such as quantum field problems in high energy physics, neutron transport as a high dimensional PDE eigenvalue problem, high frequency wave scattering in random media, and spatiotemporal estimation in air pollution modelling.