Anisotropic N-term approximation with Gaussians

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The goal of this talk is to show that a representation system based on translated, dilated and rotated Gaussians yields essentially the same N-term approximation rate for cartoon class functions as a standard curvelet or shearlet system.

The common construction principle for all these representation systems is to use one (or several) basic generating functions from which the entire system is generated by applying a countable family of affine operations to the generators. The idea of this work is to use adaptive Gaussian approximations of a given generator curvelet as new generating functions for a system of anisotropic Gaussians.

Compared to other approaches for anisotropic Gaussian expansions, as for instance presented in the work of De Hoop, Gröchenig and Romero (2014), our system is in general not a frame. We will however use a Bessel property for the normalized L^2-errors to obtain the decay rates of the N-term approximation. One advantage of the presented approach is that the necessary vanishing moments of the approximants are obtained without additional modulations of the Gaussians. Further, with minor technical adjustments our idea can be transferred from the given curvelet system to shearlet systems or more general parabolic molecules.

As one major result of this work, we will show that the described strategy yields a N-term Gaussian approximation of a cartoon class function f in which the L^2-approximation error decays as $\frac{1}{N}\log(N)^{7/2}$.

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