Optimal transport estimation by a wavelet based method

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Nowadays, optimal transport is a powerful tool used in many application domains such as economy, image processing [5], partial differential equations, etc. Particularly, the fluid mechanics formulation of the optimal transport problem proposed by Benamou and Brenier [1] reads:

$$\inf_{(\rho,m)\in C_{bb}} \int_0^1 \int_\Omega J(\rho(t,x), m(t,x))dxdt, \text{ with } J(\rho,m) = \begin{cases} \frac{|m|^2}{\rho}, & \text{if} \quad \rho > 0, \\ 0, & \text{if} \quad (\rho,m) = (0,0), \\ +\infty, & \text{elsewhere.} \end{cases}$$

where $C_{bb} = \{(\rho,m); \partial_t \rho + \nabla_x \cdot m = 0, \ \ \rho(0,.) = \rho_0 \text{ and } \rho(1,.) = \rho_1\}$ and ρ_0 is the density that we aim to push forward onto ρ_1. The functional $J(\rho,m)$ is 1-homogeneous and not differentiable. This reduces the efficiency of any gradient descent algorithm in order to solve the problem. Most prior works solve this problem using an augmented Lagrangian method [1] or an equivalent Douglas-Rachford algorithm [5], for which the projection onto C_{bb} requires the resolution of a 3D Poisson equation for 2D densities.

An alternative approach consists in working directly in the divergence-free space C_{bb}. This was done using an Helmholtz-Hodge decomposition of (ρ,m) and primal-dual algorithm in [2, 3]. To reduce the computational cost, (ρ,m) is searched on a -periodic- divergence-free wavelet basis [2]. We will provide in this work a new algorithm that encodes the solution of the optimal transportation problem with non periodic conditions, based on a new construction divergence-free wavelet basis [4]. Moreover, discretizing the Benamou and Brenier functional on this basis, we show that the numerical computation of its gradient is thus reduced to a simple fast wavelet transform. Then, an efficient gradient descent method can be used to solve the problem with a linear complexity.

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