## Viscosity solutions in practical photoclinometry

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The purpose of this study is to develop methods to estimate volumes of coal piles from satellite images.

We study and compare methods to compute a volume surrounded by a surface, supposed to be Lambertian, given its reflectance map. This is a well-known shape-from-shading problem, formalised by Horn [1] and Woodham [2] from 1970. For that purpose, we investigate the linear approximation of the Horn irradiance equation, and try to figure out how to take into account the non-linearity as a correction. However, the validity of this method depends on the illuminant direction, and can thus be discussed.

Following on from the latter considerations, we apply the viscosity solutions theory to the non-linear problem [3] [4]. An algorithm to compute evanescent viscosity solutions of the Horn equation is then proposed. We compare the results obtained with both methods : using the linear approximation (with or without non-linear correction) and using the viscosity solutions theory. Finally, we provide illustrations of our results with satellite images analysis in order to estimate volumes of coal piles, for coal is Lambertian in a good approximation.

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## References

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