A second order nonlocal variational model for crack detection on bituminous surfacing

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As observed by A. Drogoul in [5, 6], in the continuous setting, an object of very small size can be modelled by a Lebesgue set with null measure. This allows in particular to discriminate properly contours (defined as the locus of intensity discontinuities with jump), from fine structures exhibiting a discontinuity through the structure without jump.
Precisely, A. Drogoul shows, by a heuristic manner, that a filament or crack can be suitably modelled by a sequence of smooth functions with Hessian matrices blowing up in the perpendicular direction to the crack (at any point of the crack), while their gradient is null (at any point of the crack).
This observation serves as the basis of the introduced model that also handles the natural dense and highly oscillatory texture exhibited by the processed images. We propose weighting the squared Frobenius norm of the Hessian matrix of the reconstructed image function by a function whose behaviour annihilates great expansion of this norm, leading then to a variant of the elliptic approximation of the Blake-Zisserman functional [2] provided by Ambrosio, Faina and March [1].
Theoretical results including existence of minimizers, existence of a unique viscosity solution to the derived evolution problem, and a Γ-convergence result relating the elliptic functionals to the weak formulation are given. Extending then the ideas developed in the case of first order nonlocal regularization to higher order derivatives ([7]), we provide and analyze a nonlocal version of the model and an MPI implementation based on a natural domain decomposition approach.

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