# Optimal Transport for Diffeomorphic Shape Registration

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This work introduces the use of unbalanced optimal transport as a **similarity measure** for diffeomorphic matching of shape data. The similarity measure is a key object in diffeomorphic registration methods that, together with the regularization on the deformation, defines the optimal deformation – see Figure 1. Most often, similarity measures are nearly local and simple enough to be computationally fast: popular methods revolve around the computation of kernel products between measures that represent the shapes, possibly lifted into feature spaces [1].

Meanwhile, as was recently remarked in the optimal transport community, optimal transport costs can be approximated by iteratively computing kernel products in the so-called **Sinkhorn loop** [2]. We propose to take advantage of this fact to improve the robustness of kernel-related methods. As we choose appropriate feature spaces, kernel functions and number of Sinkhorn iterations (up to about 100 for convergence), we can define cheap transport-like distances that take into account features such as the shapes' curvatures.

Bottom line is that we propose a family of **fast and global** similarity measures that can be used on curves and surfaces, with possible extensions to volumetric imaging data. We shall first describe the influence of parameters and the links with other well-known distances between shapes (Hausdorff [3], kernel distances [1], standard Wasserstein and earth-mover distances [2]). Then, having combined this data attachment term with a diffeomorphic LDDMM registration routine, we will showcase results on synthetic and real data (fibres bundles, segmented brain surfaces) and show that using Sinkhorn distances increases the robustness of registration routines with respect to *large deformations*.

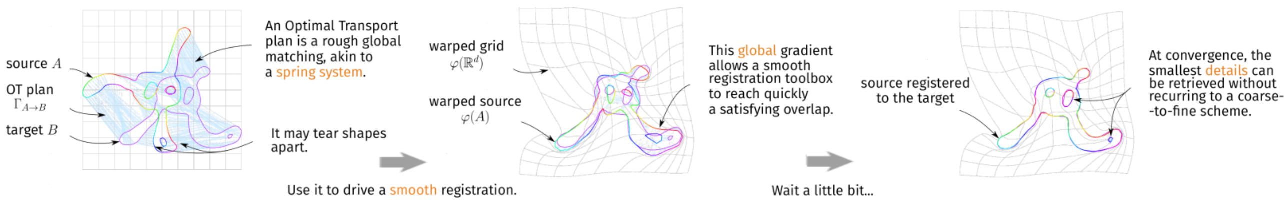


Figure 1: Diffeomorphic registration routines strive to find diffeomorphisms $\varphi$ that map a source shape $A$ to a target $B$. Most often, this is done by minimizing over $\varphi$ a cost that is the sum of a *regularization* term $\mathrm{Reg}(\varphi)$ and of a *similarity*, or *data attachment* term $\mathrm{Sim}(\varphi(A), B)$. Using for the latter a formula related to the theory of optimal transport, we are able to handle large deformations without falling into irrelevant local minimas.

**Joint work with:** Benjamin Charlier, François-Xavier Vialard, Gabriel Peyré.

## References

[1] Irene Kaltenmark, Benjamin Charlier and Nicolas Charon. A general framework for curve and surface comparison and registration with oriented varifolds, Computer Vision and Pattern Recognition (CVPR), 2017.

[2] Gabriel Peyré and Marco Cuturi, Computational Optimal Transport, `optimaltransport.github.io/book`, 2017.

[3] Nicolas Aspert, Diego Santa-Cruz and Touradj Ebrahimi. Mesh: Measuring errors between surfaces using the hausdorff distance. In Multimedia and Expo, 2002. ICME'02. Proceedings. 2002 IEEE International Conference on (Vol. 1, pp. 705-708).