Wavejets: A Local Frequency Framework for Shape Analysis

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Many shape processing methods, whether they target shape segmentation, shape denoising or shape editing, rely heavily on surface derivatives estimates. Surface derivatives are indeed useful to estimate important shape features such as normals or curvatures. The signal processing viewpoint is slightly different: instead of analyzing signal derivatives, signals are often processed by using a frequency analysis and by devising filters operating on the Fourier coefficients. We propose to bring together these two trends in a new function basis taking into account both the local surface derivatives and the angular oscillations around each point of the surface. We consider surfaces as smooth manifolds that can locally be expressed as a height field over a planar parameterization. In this setting, we extend the osculating Jets [1] in order to take into account both the local angular oscillation frequencies, and their evolution with respect to an increasing radius. This formulation, which we term Wavejets, gives valuable information on the shape.

Given a local polar coordinate system (r, θ) in the tangent plane of a sample p of a smooth surface, a Wavejet is the set of coefficients $\phi_{k,n}$ such that the heightfield describing the surface around p is written as follows:

$$f(r,\theta) = \sum_{k=0}^{\infty} \sum_{n=-k}^{k} r^k \phi_{k,n} e^{\imath n\theta} = \sum_{n=-\infty}^{\infty} \sum_{k=|n|}^{\infty} r^k \phi_{k,n} e^{\imath n\theta} \tag{1}$$

We propose to compute the Wavejets using the tangent plane for parameterization purpose and we demonstrate some theoretical properties of the Wavejets. In particular, we quantify the stability of the decomposition for a small deviation of the parameterization plane with respect to the tangent plane. This result is useful for estimating Wavejets on surfaces described by point sets. Furthermore, Wavejets coefficients can be used to compute interesting indicators of differential volumes that can be an alternative to using curvatures and further surface derivatives (see Figure 1). Those indicators are used to devise efficient surface filters. We demonstrate two applications of these filters working directly on point sets.

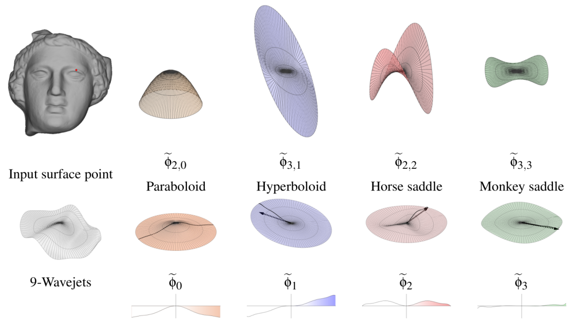

Figure 1: Wavejets decomposition around a point of a surface. Left: approximated 9-Wavejets surface. Let $\widetilde{\phi}_{k,n}(r,\theta) = r^k \left(\phi_{k,n} e^{in\theta} + \phi_{k,-n} e^{-in\theta}\right)$ and $\widetilde{\phi}_n = \sum_{k=0}^{\infty} \widetilde{\phi}_{k,n}$.

Joint work with: Julie Digne, Raphaëlle Chaine.

References

[1] F. Cazals. Estimating differential quantities using polynomial fitting of osculating jets. *Eurographics*, pages 177–187, 2003.

[2] Y. Béarzi, J. Digne, R. Chaine. Wavejets: A Local Frequency Framework for Shape Details Amplification *Computer Graphics Forum, Proc. Eurographics*, 2018