Anisotropic convolution for modeling 3D smooth shapes around 1D skeletons

A.J. Fuentes Suárez
Inria Sophia Antipolis, France
alvaro.fuentes-saurez@inria.fr

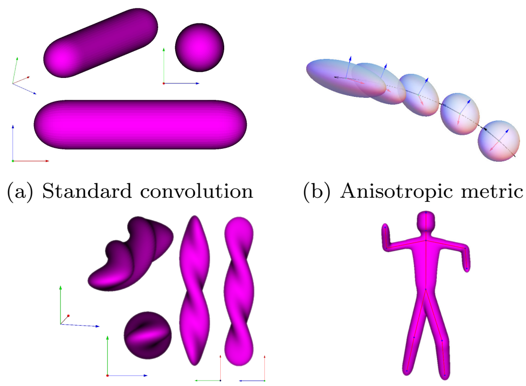
In skeleton-based modeling the user starts with a set of 1-dimensional curves that serves as skeleton for a shape surrounding it. Convolution surfaces [3] are one of those techniques that, in addition, generates smooth surfaces given by the level sets of a scalar field computed by integrating a kernel over the skeleton. This allows to construct complex surfaces with few input parameters, a limitation is that for 1-dimensional skeletons the surfaces have close-to-circular normal sections, i.e. isotropic sections (Figure 1a).

We present a new formulation for convolution surfaces around 1-dimensional skeletons that increases the variety of shapes that can be modeled by changing the isotropy of the shape around the skeleton. Let Γ be a regular curve, the new formula is then given by

$$F(x) = \int_{\Gamma} K\left(\sqrt{(x-y)^T\, G(y)\, (x-y)}\right)\, \mathrm{d}y, \qquad (1)$$

where $F : \mathbb{R}^3 \to \mathbb{R}$ is a scalar field that defines the convolution surface, $K : \mathbb{R}^+ \to \mathbb{R}^+$ is a kernel function satisfying $K(x) > 0 \implies K'(x) < 0$, and G is a function that assigns a positive definite symmetric matrix to each point on the skeleton Γ. Standard convolution surfaces are particular cases of (1) with G the identity matrix.

In order to increase the modeling freedom we define $G = HDH^T$ where H is a moving frame on Γ, and D is a positive diagonal matrix. This allows us to define a varying metric on Γ (Figure 1b) that effectively changes the shape of the final surface (Figure 1c), and yields anisotropic normal sections. Although we use 1-dimensional skeletons, the flattened shapes we are able to get can mimic the shapes of convolution surfaces around 2-dimensional skeletons (Figure 1d).

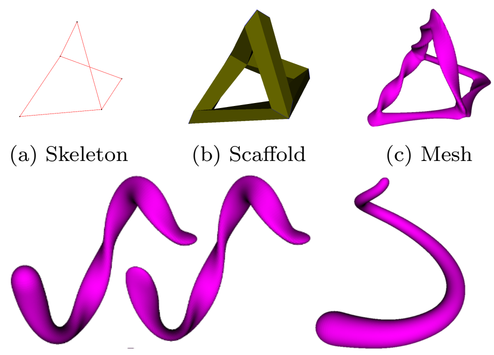


(a) Standard convolution (b) Anisotropic metric (c) Anisotropic convolution (d) Planar shapes

Figure 1: Extended modeling capabilities with anisotropic convolution.

We introduce spatial $\mathcal{G}^1$ circular splines as the preferred approximation to the skeleton (Figure 1d). Compared to line segment approximations, this approximation decreases the number of pieces used for generating curved surfaces. It is also the basis for a meshing technique based on ray shooting and scaffolding [2] that generates a quad dominant mesh for the convolution surface that follows the structure of the skeleton (Figure 2). We prove [2] that a scaffold mesh can always be constructed, even in the presence of cycles in the skeleton.

We implemented anisotropic convolution and the scaffolding technique for $\mathcal{G}^1$ skeletal curves as a C++ library. The polygonization of the surfaces is given by a mesh containing only quads except perhaps at extremities of the model (as in [1]).



(a) Skeleton (b) Scaffold (c) Mesh

(d) Anisotropic convolution around $\mathcal{G}^1$ skeletons

Figure 2: Anisotropic convolution surface polygonization with a scaffolding technique.

Joint work with: E. Hubert. *Inria Sophia Antipolis, France*

References

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[3] Cédric Z. *Skeleton-based Implicit Modeling & Applications*. Phd, Université de Grenoble, 2013.