Discrete curvatures and solution of biomembranes

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The so-called (Helfrich) biomembrane problems asks for surface with a prescribed genus that solves the variational problem:

$$\min_S W[S] := \int_S H^2 \, dA \text{ s.t. } \begin{cases} A[S] := \int_S 1 \, dA = A_0, \\ V[S] := \frac{1}{3} \int_S [x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}] \cdot \hat{\mathbf{n}} \, dA = V_0, \\ M[S] := - \int_S H \, dA = M_0. \end{cases}$$

The physical relevance and mathematical depth make it a challenging benchmark problem for applied geometers. We present three contributions to the numerical treatment of this and related problems:

- We show that certain *conforming* numerical methods, using techniques developed in our community such as subdivision surfaces or manifold splines, are convergent. Such a convergence analysis has been lacking in the literature.
- We prove that several *non-conforming* numerical methods based on a number of well-known discrete curvature operators on PL surfaces fail to converge.
- We provide numerical evidences that techniques in conformal geometry can be used to regularize the non-conforming PL methods, giving a new method that produce results consistent with those obtained from conforming methods. We formulate a number of open questions.

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