Quasi-interpolation methods based on cubic Powell–Sabin splines

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The process of constructing bivariate polynomial splines on triangulations can be simplified by applying a Powell–Sabin refinement [4] to a general triangulation. Recently, C^1 cubic splines on Powell–Sabin triangulations with and without additional smoothness constraints have been considered for possible use in approximation theory and geometric modelling [2, 3, 5]. Also, a stable B-form for such splines, which is based on local basis functions that form a convex partition of unity, has been provided [1]. This B-form is interesting also because it allows a representation of classical C^1 quadratic Powell–Sabin splines and C^1 cubic Clough–Tocher splines in a unified context.

In this talk we make use of the cubic Powell–Sabin B-form to introduce a general framework of methods for constructing quasi-interpolation operators based on local polynomial approximation. We assign a linear functional to each basis function to specify the coefficients in the B-form. The functionals have a standard form, i.e. they take a cubic polynomial and evaluate it using the blossoming principle. Using this approach, a quasi-interpolation operator can be defined by providing a collection of cubic polynomials, which can be constructed based on local data sites using any standard approximation method, e.g. Lagrange or Hermite interpolation, least square approximation, etc. We study the properties of such quasi-interpolation operators and present general recipes to specify them in a way that they have a global or local polynomial precision and satisfy certain additional smoothness constraints. Finally, we derive some concrete methods and compare them with numerical experiments.

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