Polynomial approximation with convex constraints

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We consider the problem of building a polynomial approximation with special cases of convex constraints [1]. The class of constraints we consider are inequalities linear in the polynomial expansion coefficients and includes, for example, positivity, boundedness, and/or monotonicity constraints. We reformulate this problem as a convex optimization problem on a high-dimensional sphere and devise efficient numerical methods to solve the problem using the spherical geometry.

We show that when the expansion basis elements are univariate orthonormal polynomials, then the convex set on the sphere satisfies special properties and geometric structure that can be exploited in an algorithm. We devise two-step schemes that can ensure satisfaction of the constraints, even though the geometry of the convex set is only implicitly defined. The first step discretizes the spherical set so constraints can be approximated as finitely many linear equalities. The second step uses an initial guess generated from the first step in a type of projections onto convex sets algorithm [2] on the sphere. The spherical formulation of the algorithm results in very stable computations, even when very high degree polynomials on unbounded domains are considered. We apply our approach to several numerical examples, investigating order of convergence versus regularity, and consider our scheme in relevant finite element methods where positivity or monotonicity is required for the solution.

References

[1] J.R. Rice. Approximation with Convex Constraints. *Journal of the Society for Industrial and Applied Mathematics*, 11(1):15-32, 1963.

[2] H.H. Bauschke and J.M. Borwein On Projection Algorithms for Solving Convex Feasibility Problems. *SIAM Review*, 38(3):367-426, 1996.