Multivalued support function at inflection points of planar curves

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We study the behavior of the support function (see e.g. [1, 2]) in the neighborhood of a curve inflection. The gauss map at the inflection point is not regular and in the neighborhood is typically not injective. The support function is thus not regular and typically multivalued. We describe this function using an implicit algebraic equation and the Puiseux series of its branches. We show the correspondence between the degree of the approximation of the primary curve (using Taylor series) and the degree of the approximation of the support function (using Puiseux series). Based on this results we are able to approximate curve with inflections by curves with a simple support function which consequently possess rational offsets. We also analyze the approximation degree of this kind of dual approximation. In particular, we explain why the approximation order of the $C^1$ Hermite interpolation drops from 4 to 3 when an inflection occurs. Such a phenomenon was experienced e.g. when using segments of hypocycloids or epicycloids, see [3]. We propose an alternative adaptive subdivision scheme, which ensures the approximation degree 4 both for the inflection–free segments and the segments with inflections.

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References