Finsler Metrics for Fronts Propagation and Active Contours Evolution

Da Chen[1,2] and Laurent D. Cohen[1]
[1]CEREMADE, University Paris Dauphine, PSL Research University
[2]Hôpital des Quinze-Vingts, Paris, France
{chenda,cohen}@ceremade.dauphine.fr

Fronts propagation can be efficiently carried out through solutions to the Eikonal partial differential equation (PDE). It has been widely applied to image analysis thanks to its solid mathematical background and the well-established Eikonal solvers such as the fast marching method [1]. There exist two popular schemes which can be used for image segmentation. The first one is an interactive scheme which requires user-provided scribes to indicate foreground and background (see [2] for a review of the existing related literature), while the second one is the dual-front scheme [3] for active contours evolution. Both image segmentation schemes are based on the Voronoi diagram and Voronoi regions, for which the common objective is to seek the boundary of two adjacent Voronoi regions. In the context of geodesic framework, the Voronoi regions can be identified from the geodesic distance map which is the solution to the Eikonal PDE.

The geodesic metric plays a crucial role for the computation of geodesic distance by solving the Eikonal PDE. Existing fronts propagation models only invoke metrics with a symmetric Riemannian form, which cannot take into account the asymmetry information to help the fronts propagation. In order to solve this problem, we consider a Finsler geodesic metric $\mathcal{F} : (x,\nu) \in \Omega \times \mathbb{R}^n \mapsto \mathcal{F}(x,\nu) \in [0,\infty]$ for geodesic distance computation, where Ω is the image domain. A Finsler metric $\mathcal{F}$ is said to be asymmetric if $\exists x \in \Omega$ and $\exists \nu \in \mathbb{R}^n$ such that the inequality $\mathcal{F}(x,\nu) \neq \mathcal{F}(x,-\nu)$ is held. This means that some asymmetric enhancements, which are dependent of the image data or the evolutional contour, can be considered during the geodesic distance computation.

For both image segmentation schemes, either interactive scheme or dual-front scheme, the Finsler geodesic metrics are constructed through a feature vector field $\mathfrak{p} : \Omega \to \mathbb{R}^n$. The principle for Finsler metric $\mathcal{F}$ is that for a fixed point x near the object boundaries, $\mathcal{F}$ should satisfy the inequality $\mathcal{F}(x,-\mathfrak{p}(x)) > \mathcal{F}(x,\mathfrak{p}(x)) > \mathcal{F}(x,\mathfrak{p}(x)^\perp)$, where $\mathfrak{p}(x)^\perp$ is the orthogonal vector of $\mathfrak{p}(x)$. In this case, when the fronts tend to cross an edge, the Finsler metric will produce a very large geodesic distance value so as to reduce the risk of fronts leaking problem. In the following we briefly introduce how to compute feature vector field.

Interactive segmentation scheme. For interactive segmentation, the feature vector field $\mathfrak{p}$ can be derived from the gradient vector flow [4] and the norm of the image gradients. In this case, a feature vector $\mathfrak{p}(x)$ points to an edge from the current position x, thus providing asymmetric information for the Finsler metric. When x is near an edge, the value of $\mathfrak{p}(x)$ will be large to slow down the fronts propagation.

Dual-front scheme. During the active contours evolution, let $\mathcal{C}$ be the current contour. The objective is to seek a new contour $\mathcal{C}_{\text{new}}$ that is located at the zero-crossing locations of the shape gradient of a given region-based similarity measure. Thus, we construct the feature vector field $\mathfrak{p}$ through the normal field of the contour $\mathcal{C}$ and the shape gradient involving its magnitudes and signs.

References

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