Rigid–body motion interpolation using cubic PH biarcs

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Polynomial Pythagorean–hodograph (PH) curves in space ([1]), which are characterized by the property that the unit tangent is rational, have many important features for practical applications. One of them is that these curves can be equipped with rational orthonormal frames called Euler–Rodriques (ER) frames, where the first frame vector coincides with the unit tangent. The second important property is that the arc–length function is a polynomial. Joining these two properties we can construct motions of a rigid–body that interpolate some given positions and have a prescribed length of the center trajectory.

In the talk an interpolation scheme for G^1 Hermite motion data, i.e., interpolation of data points and rotations at the points, with cubic PH biarcs is presented, where the rotational part of the motion is determined by the ER frame. Further, the length of the biarc is interpolated too. It is shown that the solution exists for any data and any length greater than the difference between the interpolation points. Moreover, the interpolant depends on some free parameters, which can be chosen so that the center trajectory is of a nice shape and the rotation of vectors in a normal plane around the tangent is minimized. The derived theoretical results are illustrated with numerical examples.

References

[1] R. T. Farouki. Pythagorean–Hodograph Curves: Algebra and Geometry Inseparable. *Geometry and Computing*, vol. 1. Springer, Berlin, 2008.