Bivariate polynomial interpolation on interlacing lattices

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The question of whether polynomial interpolation in two variables is well defined, or 'unisolvent', depends not only on the number of points but on their positions. For polynomial degree n we need $N = (n+2)(n+1)/2$ points, i.e., 3 points for degree 1, 6 points for degree 2, 10 points for degree 3, and so on. Two well-known classes of point configurations for which interpolation is unisolvent are the *principal lattice*, a triangular grid, and the *natural lattice*, the intersections of non-parallel lines.

In this talk I will discuss the *interlacing lattice* and why it too is unisolvent. This lattice is the union of two interlacing rectangular grids, one square, the other quasi-square. For example, for degree 1, the quasi-square grid is 2×1 and the square one is 1×1. For degree 2, the square grid is 2×2 and the quasi-square one is 2×1. For degree 3, the quasi-square grid is 3×2 and the square one is 2×2, and so on. This lattice includes the so-called Padua points, generated by Lissajou curves, as a special case.

Unisolvence is due to the non-singularity of the leading principal submatrices of certain univariate divided difference matrices. I will try to give some simple examples of this.