A forward mapping approach for rendering free-form curves and surfaces

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Curves and surfaces constructed by transcendental functions only or by transcendental functions together with polynomials have distinguished properties and have been studied extensively in the latest few decades; see for example [1, 2]. Unlike polynomial based curves and surfaces such as Bézier, B-spline and NURBS curves and surfaces which can be rendered efficiently by forward differencing techniques [3, 4], curves and surfaces constructed by transcendental functions have to be evaluated by loading special mathematical libraries.

It is observed that spaces spanned by basis functions for curves and surfaces in CAGD are usually closed with respect to a differential operation. Therefore, free-form curves constructed by control points and basis functions can be formulated as the solutions of linear differential systems [5]. Besides free-form curves, we further show that free-form surfaces constructed by control points and basis/blending functions and curves on a surface with skew parametrization can also be represented as the solutions of linear differential systems. From this point of view, we show that points lying on a free-form curve or surface curve, iso-curves lying on a surface can all be obtained by forward mapping of prior points or prior iso-curves.

Particularly, the forward mapping matrices are formulated as the exponentials of constant coefficient matrices and the selected parameter steps. Since there are no always satisfying methods to compute the exponentials of matrices, we compute the forward mapping matrices for curves and surfaces based on elementary decomposition of basis functions. We noticed that bases of free-form curves and surfaces are usually obtained by compositions or tensor products of a few elementary basis functions such as polynomials, trigonometric functions or hyperbolic functions, etc. Based on some identities of the elementary basis functions, efficient algorithms are developed to compute the forward mapping matrices. By forward mapping techniques, free-form curves and surfaces constructed by algebraic as well as transcendental functions can be evaluated and rendered efficiently with only arithmetic operations.

References

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