# Accuracy Bounds for Super-Resolution

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The problem of mathematical super-resolution is the problem of extrapolating the fine details of a signal $F$ from band limited and noisy measurements of its Fourier transform, given a priori that the signal is sparse.

We consider the reconstruction of a spike-train signal class of the form $F(x) = \sum_{j=1}^{d} a_j \delta(x - x_j)$. It is well known that the reconstruction of this class captures the difficulty of the super-resolution problem, extends to other models, and presents an important problem in Mathematics and Engineering (see e.g. [7, 5]).

We address the case where some of the nodes of $F$ nearly collide. As a result, the inversion is highly singular and known to present major mathematical difficulties.

Assume that the Fourier transform of $F$, $\mathcal{F}(F)(s)$, is known for $s \in [-B, B]$ with an absolute error not exceeding $\varepsilon > 0$. The subject of this talk, following the work in [6, 2, 1] and recent results, is to give accuracy bounds on the stability of the reconstruction when $l \leq d$ nodes of $F$ form a cluster of size $h << \frac{1}{B}$ while the rest of the nodes are well separated.

We show lower and upper bounds on the error rate in this case, both of order $(Bh)^{-2l+1}\varepsilon$.

Our approach is based on forming and analysing certain Prony systems, which appear via an appropriate sampling of the Fourier transform. We use the analysis of the geometry of error amplification in moment reconstruction of spike-train signals, carried out in [1].

The above result follows the work done in [6, 2, 1, 3, 4]. The derived lower bound is the same as shown in [2, 6], in the latter the nodes are restricted to a grid, but proved using a different technique.

The upper bound, on the other hand, *closes a gap* between the lower and upper bounds of both [6, 2], for the case where $l < d$ nodes form a small cluster while rest of the nodes are well separated. By those results the lower bound is of order $(Bh)^{-2l+1}\varepsilon$ while the upper bound is of order $(Bh)^{-2d+1}\varepsilon$. Indeed it is shown that the cluster nodes play the center role in determining the reconstruction accuracy.

**Joint work with:** Yosef Yomdin.

## References

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