Spline-based approximation of smooth functions via F-transform

Martins Kokainis
University of Latvia
martins.kokainis@lu.lv

This contribution considers approximation of smooth functions via F-transform (see [1]), which consists of two stages: the direct transform, which maps a continuous function to a vector, and the inverse transform, which maps the vector to a function, approximating the initial function. In [2] the approach was generalized so that the vector given by the direct transform has degree-m polynomial components (the F^m-transform). By construction the inverse transform of such vector yields a function which coincides with the initial function whenever the latter is a polynomial of degree-m and approximates the initial function under some smoothness assumptions. The F-transform method has found several applications in various fields, e.g., images processing, time series analysis and non-parametric regression.

The key object in the F-transform is the fuzzy partition, i.e., the partition of the given interval into fuzzy sets. We have shown [3] that uniform partition by odd-degree B-splines allows to improve the approximation quality by the inverse F^m-transform. Namely, we prove that using B-splines of degree at least $(2m+1)$ allows to achieve the following:

- the inverse F^m-transform coincides with the initial function whenever it is a polynomial of degree at most $2m+1$;
- the inverse F^m-transform of a function $f \in C^{2m+2}[a,b]$ approximates f pointwise with order of approximation $O(h^{2m+2})$.

In this proposal we discuss approximation of $f \in C^{2m+2}[a,b]$ (uniformly in $[a,b]$ with order $O(h^{2m+2})$) by the B-spline based inverse F^m-transform, uniform approximation of the derivatives of f by the respective derivatives of the inverse F^m-transform of f (with corresponding loss in the approximation order), applications to solving linear boundary value problems, and briefly sketch extensions to the multivariate case.

Joint work with: Svetlana Asmuss (University of Latvia).

References

[1] I. Perfilieva. Fuzzy transforms: theory and applications. *Fuzzy Sets and Systems*, 157, 993–1023, 2006.

[2] I. Perfilieva, M. Daňková, B. Bede. Towards a higher degree F-transform. *Fuzzy Sets and Systems*, 180, 3–19, 2011.

[3] M. Kokainis, S. Asmuss. Continuous and discrete higher-degree F-transforms based on B-splines. *Soft Computing*, 21(13), 3615–3639, 2017.