Stabilized FEM for Simplex Space-Time Meshes in 4D

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A space-time finite element discretization, by means of 4D simplex space-time elements, referred to as pentatopes [1], leads to completely unstructured grids with varying levels of refinement both in space and in time. Another advantage of space-time meshes is the capability of connecting different spatial meshes at the bottom and top levels of the space-time slab and coping with complicated domain movements/rotations that the standard ALE techniques cannot resolve without remeshing.

The objective of this talk is to emphasize how the 4D space-time finite element discretization within the framework of local time-stepping is applied to mold filling cases, as discussed for simple benchmark cases in [2]. Their numerical simulation based on computational fluid dynamics is tremendously demanding and accounts for the two-phase flow, the propagation of an interface, the surface tension, discontinuities in material properties, etc. Nevertheless, the efficiency of our simulations is improved when using adaptive temporal refinement in the vicinity of the evolving front. The numerical examples, used for validating the unstructured space-time mesh solver and the refinement scheme, involve the filling of different mold cavities.

The transient, incompressible Navier-Stokes equations describe the two-phase flow during the mold filling stage, while assuming that the fluids of our interest are incompressible and Newtonian. The level-set method is used to capture the moving front, due to its inherent ability to handle severe topological changes of the interface shape. We use a stabilized space-time finite element method in order to discretize the equations. The stabilization parameter is defined based on the contravariant metric tensor, as shown in [3]. Its definition is extended in 4D and allows us to deal with complex anisotropic simplex meshes in the space-time domain.

Joint work with: Lutz Pauli, Max von Danwitz, Marek Behr.

References

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