Distance from a point to a parametric surface

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Parametric curves and surfaces, including Bézier curves and tensor-product Bézier surfaces are widely used in geometric modelling and CAD. Distance from a point p to a parametric surface $\mathcal{S} \in \mathbb{R}^3$ have been studied in numerous applications such as intersection of surfaces, surface fitting, offset surface, e.g. [4].

In order to find closest points on $\mathcal{S}$ to $p \in \mathbb{R}^3$, one considers orthogonal projections q of p onto $\mathcal{S}$. The points $q \in \mathcal{S}$ which minimize the distance function from p to $\mathcal{S}$ are among all the orthogonal projections of p onto $\mathcal{S}$. The extrema values of square distance function which are candidates to be closest points, can be solved by iterative methods such as Newton-Raphson, by subdivision such as [4] or by algebraic methods such as [1],[3]. It is well-known that there are inital value and convergence problems for iterative methods. Also, they cannot detect points with several closest points. Subdivision methods are more robust, because they do not require any initial guess. Moreover, usage of additional elimination criteria allows subdivision methods to compute only the needed solutions [4].

On the other hand, there are also algebraic methods which rely on symbolic computation. For instance, the methods which are based on study of Jacobian matrix obtained from the square distance function, followed by Groebner basis computation, use algebraic techniques [3]. Besides, there are other algebraic methods, namely symbolic-numeric methods, based on matrix representations, such as [1], which require still symbolic computation. We introduce a new algebraic method, more precisely a symbolic-numeric method, which allows using numerical approximations, i.e. floating-point data relying on numerical linear algebra.

The method in [1] uses the moving surfaces that have been introduced by Sederberg and Chen in [2] to find the q's. Two moving surfaces defined by two polynomials in one variable of extremely high degree yields the q's. Each one is in one of variables parameterizing $\mathcal{S}$. Thus, the 2 intermediary matrices $\mathcal{M}_1, \mathcal{M}_2$ from which moving surfaces are obtained are considerably big. Already for biquadratic Bézier surfaces, the method requires lots of memory usage and computations. For bicubic Bézier surfaces, computations become impossible to handle.

We introduce a new method for the closest points problem based on algebraic techniques. We first write a parametrization $\mathcal{P}$ for the normal lines (resp. planes) to a given parametric surface (resp. curve) in tensor-product rational Bézier surface form. Then, differently from [1], we proved that it is possible to construct only one moving surface but in multi-graded context, hence in a much smaller degree. Thus, a smaller elimination matrix $\mathcal{M}$ is constructed. Its proof is based on a fine analysis of the syzygies of $\mathcal{P}$. After that, the corank of the nullspace of $\mathcal{M}$ gives the number of the q's on $\mathcal{S}$ for a general point $p \in \mathbb{R}^3$. More precisely, until this step the method does not depend on the chosen p. Finally, after evaluating $\mathcal{M}$ at a given p, generalized eigenvalues and eigenvectors computation allows to obtain the coordinates of orthogonal projections of p onto $\mathcal{S}$. Here, we rely on numerical linear algebra in order to deal with floating-point data. Also, the method does not miss any orthogonal projections which are all closest points on $\mathcal{S}$ to p. Lastly, among all the q's on $\mathcal{S}$, which minimize the distance function gives the distance from p to $\mathcal{S}$.

Joint work with: Nicolàs Botbol, Laurent Busé, Marc Chardin

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