Smoothness concepts for polyhedral surfaces

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The approximation of a freeform surface by a polyhedral surface is an important topic in many applications. It is particularly subtle in architecture where vertices, edges and faces are often highly visible, and any notion of fairness or smoothness must take new aspects into account. In this talk, we will present an overview of recent results on smoothness concepts for polyhedral surfaces [1, 2] and point to ongoing unpublished research.

One way of defining smoothness of a polyhedral surface uses the Gaussian images of its vertex stars. One requires that these image are free of self-intersections. If they are in addition star-shaped, one obtains a behavior which is very similar to that of smooth surfaces in classical differential geometry. We come up with suitable notions of tangent planes at vertices, asymptotic directions, and parabolic curves that are invariant under projective transformations. It is remarkable that seemingly mild conditions significantly limit the shapes of faces of a smooth polyhedral surface.

The concepts will be illustrated by standard triangle, quad and hex meshes, but also by polyhedral surfaces with the combinatorics of a semi-regular tiling [3]. These polyhedral patterns exhibit an interesting phenomenon: Some of the patterns adapt the shapes of tiles according to the curvature of an underlying smooth reference surface while others lack this behavior. However, this second type leads to rougher polyhedral surfaces which one would not like to classify as smooth ones. Thus, polyhedral patterns provide good examples for testing the suitability of smoothness concepts for polyhedral surfaces.

Joint work with: F. Günther, C. Jiang, C. Tang, A. Vaxman, J. Wallner, P. Wonka.

References

[1] F. Günther, C. Jiang, and H. Pottmann. Smooth polyhedral surfaces. *ArXiv e-print*, 1703.05318, 2017.

[2] C. Jiang, F. Günther, J. Wallner, and H. Pottmann. Measuring and controlling fairness of triangulations. *Advances in Architectural Geometry 2016*, pp. 24-39.

[3] C. Jiang, C. Tang, A. Vaxman, P. Wonka, and H. Pottmann. Polyhedral patterns. *ACM Trans. Graphics*, 34(6):#172,1–12, 2015.