The approximation of the surfaces areas using classical Bernstein quadrature formula

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The Bernstein polynomials $B_n(f;x) = \sum_{k=0}^n {n \choose k} x^k (1-x)^{n-k} f\left(\frac{k}{n}\right)$ have open a new era in approximation theory starting with the year 1912, when S.N. Bernstein presented his famous proof of the Weierstrass approximation theorem and continuing with thousands of interesting papers until today. Thanks to some important properties as uniform approximation, shape preservation and variation diminishing, Bernstein polynomials are indispensable tools in computer aided geometric design, as well as in other areas of mathematics.

The approximation of functions by Bernstein polynomials is made on the interval [0, 1]. Thinking at practice utility of Bernstein polynomials, it is easy to remark that approximation of functions defined on the interval [a, b], where a, b being real and finite number, excepting a := 0, b := 1 appears more often than approximation of functions defined on the interval [0,1]. Taking the above sentences into account, we present the classical Bernstein polynomials [1], [2] defined for any $x \in [a, b]$, by

$$B_n(F;x) = \frac{1}{(b-a)^n} \sum_{k=0}^n \binom{n}{k} (x-a)^k (b-x)^{n-k} F\left(a + \frac{k(b-a)}{n}\right).$$
(1)

In the present talk, we want to highlight an applicative side of classical Bernstein polynomials, in contrast to the well-known theory of the uniform approximation of functions. An example in this sense could be the approximation of various surfaces areas by using the classical Bernstein formula, given by

$$\int_{a}^{b} F(x)dx = \frac{b-a}{n+1} \sum_{k=0}^{n} F\left(a + \frac{k(b-a)}{n}\right) - \frac{(b-a)^{3}}{12n} F''(\xi), \text{ with } \xi \in (a,b).$$
(2)

Taking n = 1 in the above relation (2) we get "surprisingly" the trapezoidal formula

$$\int_{a}^{b} F(x)dx = \frac{b-a}{2}(F(a) + F(b)) - \frac{(b-a)^{3}}{12}F''(x), \text{ with } \xi \in (a,b).$$

In order to find an exact place on the map of closed Newton-Cotés quadrature formulas, for this new approximation formula of surfaces areas, we make the following notations

$$\int_{a}^{b} F(x)dx \approx \int_{a}^{b} B_{n}(F;x)dx = \frac{b-a}{n+1} \sum_{k=0}^{n} F\left(a + \frac{k(b-a)}{n}\right) =: I_{n}[F],$$
(3)

which we call the classical Bernstein quadrature formula and

$$|R_n[F]| := \left| \int_a^b R_n(F;x) dx \right| \le \frac{(b-a)^3}{12n} M_2[F], \text{ where } M_2[F] := \max_{\xi_x \in [a,b]} |F''(\xi_x)|,$$

the upper bound estimation of appropriate remainder in classical Bernstein formula (2).

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References

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