Geometry and motion using geometric algebra

Glen Mullineux
University of Bath, UK
g.mullineux@bath.ac.uk

There are a number of ways of representing and manipulating geometric transforms. In recent years quaternions and dual quaternions [1] have become increasingly used. These ideas have been extended to the use of geometric algebra of which there are a number of variations [2, 3]. In particular, these approaches allow rigid-body transforms to be represented exactly and robustly. Consideration of (smoothly) varying transformations enables motions of rigid bodies to be generated.

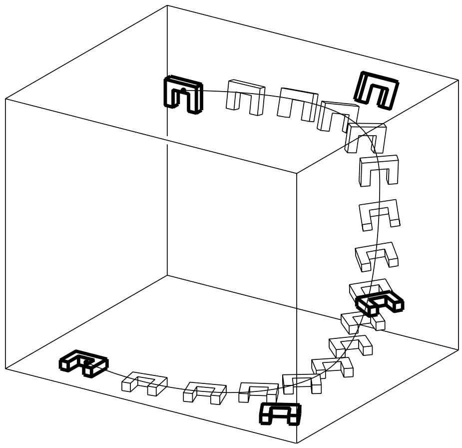

Fig 1: Quadratic additive B-spline motion defined by five control poses

Techniques such as slerp (spherical linear interpolation) [4] allow transformations to be combined multiplicatively. They can also be combined additively which, although perhaps less natural, removes the need to deal with logarithms and exponentials. Such pairwise combinations can be used as part of the de Casteljau algorithm to allow free-form motions to be created from a small number of control poses. Figure 1 shows a quadratic B-spline motion with five control poses. Similarly, a number of results and techniques for free-form curves and surfaces pass over to free-form motions.

Dual quaternions and many forms of geometric algebra allow such constructions to be undertaken. The suitability of any such approach can depend upon a number of other factors associated with any given application. However, it is suggested that what is often required in the ability to represent 3D geometry (Euclidean or projective) in a natural way, and the ability to apply rigid-body transforms in a straightforward way to this geometry.

A small number of existing approaches are considered with respect to these requirements, including a version of geometric algebra used successfully by the authors [5, 6]. In particular, its uses in dealing with a problem in geometry and with free-form motions are discussed and illustrated.

Joint work with: Ben Cross, Robert J. Cripps, University of Birmingham, UK.

References

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