Optimal transport between a simplex soup and a point cloud and applications

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We propose a numerical method to find the optimal transport map between a measure supported on a lower-dimensional subset of $\mathbb{R}^d$ and a finitely supported measure.

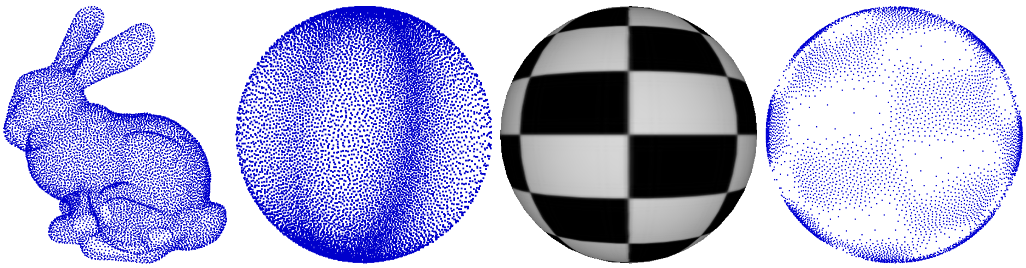
More precisely, the source measure is assumed to be supported on a simplex soup, i.e. on a union of simplices of arbitrary dimension between 2 and d. As in [1], we recast this optimal transport problem as the resolution of a non-linear system where one wants to prescribe the quantity of mass in each cell of the so-called *Laguerre diagram*. We prove the convergence with linear speed of a damped Newton's algorithm to solve this non-linear system. The convergence relies on two conditions:

1. a genericity condition on the point cloud with respect to the simplex soup,
2. a (strong) connectedness condition on the support of the source measure defined on the simplex soup.

As in for Newton-based methods, the choice of the initial guess is important. We describe and prove the correctness of three different procedures to find such initial guess. We also show the pros and cons of each one on different settings.

We then apply our algorithm in $\mathbb{R}^3$ to compute optimal transport plans between a measure supported on a triangulation and a discrete measure. We also detail some applications such as optimal quantization of a probability density over a surface, remeshing or rigid point set registration on a mesh.

Finally, we show that we can use this algorithm to solve problems arising in non-imaging optics (optimal transfer of light between a source and a target).



Optimal quantization of triangulated surfaces for different densities and surfaces.

Joint work with: Quentin Mérigot, Boris Thibert.

References

[1] Franz Aurenhammer, Friedrich Hoffmann, and Boris Aronov. Minkowski-type theorems and least-squares clustering. *Algorithmica*, 20(1):61–76, 1998.

[2] Quentin Mérigot, Jocelyn Meyron, and Boris Thibert. An algorithm for optimal transport between a simplex soup and a point cloud. *arXiv preprint arXiv:1707.01337*, 2017.