Control of curvature extrema on curves and surfaces

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Relations between fairness and curvature were already exploited in a collection of papers edited by Sapidis [1]. Furthermore, according to Farin [2]: "*A curve is fair if its curvature plot is continuous and consists of only a few monotone pieces*". This implies that such a curve needs to have a low number of curvature extrema. The currrent work, in line with [3], explores new alternatives to control curvature extrema on curves and surfaces.

The first objective is to transform shapes without introducing new curvature extrema. Using Lie sphere transformations [4], there are 8 degrees of freedom to transform a solid shape without introducing new curvature extrema. Examples of Lie sphere transformations of ellipsoids are shown in Figure 1.

Following the work of Pottmann [5], optical geometry allows taking a step further towards more general transformations. An important link between the differential properties of functions and the differential properties of surfaces can then be established. In the plane, the zeros of the third derivative of a function that represents a mirror correspond to the extrema of curvature of a reflected wavefront. This result can be generalized to surfaces. Porteous [7] formalism of differential forms is used for this purpose.

The second objective is to selectively erase curvature extrema, remaining as close as possible to an initial geometry. For curves this can be peformed using B-spline optimization with constraints [6]. For general surfaces, the problem seems to be very difficult. On the other hand, focusing on more specific surfaces, it is possible to make some progress. Famillies of canal surfaces with controled curvature extrema are obtained.

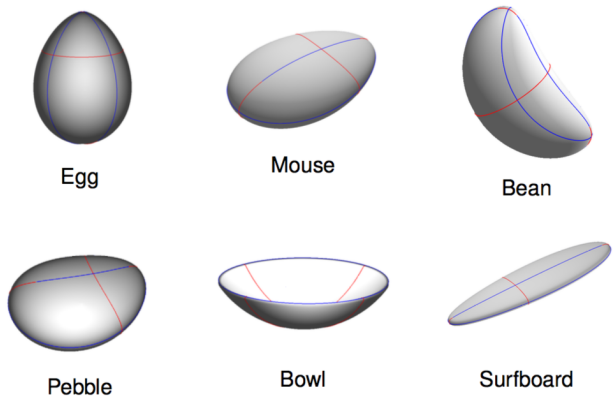

Figure 1: Lie sphere transformations of ellipsoids with lines of curvature extrema (ridges) drawn on the surfaces

Joint work with: François Guibault, Christophe Tribes and Jean-Claude Léon.

References

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