# Identification problems in multi-exponential analysis

Annie Cuyt and Wen-shin Lee
University of Antwerp
wen-shin.lee@uantwerpen.be

We consider the interpolation of a $d$-variate exponential sum

$$f(x_1, \ldots, x_d) = \sum_{j=1}^{n} \alpha_j \exp(\phi_{j,1} x_1 + \cdots + \phi_{j,d} x_d).$$

In the univariate case, where $d = 1$, there is an entire branch of algorithms, which can be traced back to Prony's method dated in the 18th century and devoted to the recovery of the $2n$ unknowns, $\alpha_1, \ldots, \alpha_n, \phi_1, \ldots, \phi_n$ in

$$f(x) = \sum_{j=1}^{n} \alpha_j \exp(\phi_j x).$$

In the multivariate case, where $d > 1$, it remains an active research topic to identify and separate distinct multivariate parameters from results computed by a Prony-like method from samples along projections.

On top of the above, if the $\phi_{j,k}$ are allowed to be complex, the evaluations of the imaginary parts of distinct $\phi_{j,k}$ can also collide. This aliasing phenomenon can occur in either the univariate or the multivariate case.

Our method interpolates $f(x_1, \ldots, x_d)$ from $(d+1) \cdot n$ evaluations [1]. Since the total number of parameters $\alpha_j$ and $\phi_{j,k}$ is exactly $(d+1) \cdot n$, we interpolate $f(x_1, \ldots, x_d)$ from the minimum possible number of evaluations. The method can also be used to recover the correct frequencies from aliased results [2]. Essentially, we offer a scheme that can be embedded in any Prony-like algorithm, such as the least squares Prony version, ESPRIT, the matrix pencil approach, etc., thus can be viewed as a new tool offering additional possibilities in exponential analysis.

## References

[1] A. Cuyt, W.-s. Lee. Multivariate exponential analysis from the minimal number of samples. *Advances in Computational Mathematics*, to appear.

[2] A. Cuyt, W.-s. Lee. How to get high resolution results from sparse and coarsely sampled data. *arXiv:1710.09694 [math.NA]*, 2017.