Greedy measurement selection for state estimation

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Inspired by the problem of approximating solutions of parametric PDEs where the solution depends smoothly on a high or infinite dimensional parameter, we are interested in approximating an element u which is known to lie on a smooth compact manifold $\mathcal{M}$ in an ambient Hilbert space V. We can approximate the manifold by a linear space V_n, where n is of moderate dimension such that $\mathcal{M}$ is never too far from V_n, that is, $\operatorname{dist}(u, V_n) \leq \varepsilon$ for any $u \in \mathcal{M}$. Otherwise, our only assumed knowledge of u is m observed linear measurements of u of the form $\ell_i(u) = \langle w_i, u \rangle$, $i = 1, \dots, m$, where $\ell_i \in V'$ and w_i are the Riesz representers, and we write $W_m = \operatorname{span}\{w_1, \dots, w_m\}$.

In this setting, the observed measurements and V_n can be combined to produce an approximation u^* of u up to accuracy

$$\|u - u^*\| \leq \beta^{-1}(V_n, W_m)\, \varepsilon$$

where

$$\beta(V_n, W_m) := \inf_{v \in V_n} \frac{\|P_{W_m} v\|}{\|v\|}$$

plays the role of a stability constant. For a given V_n, one relevant objective is to guarantee that $\beta(V_n, W_m) \geq \gamma > 0$ with a number of measurements $m \geq n$ as small as possible.

Assuming that the measurement functionals ℓ_i belong to a complete dictionary, we propose and study certain a algorithm for their selection. This algorithm may be viewed as a "collective" version the well-known orthogonal matching pursuit (OMP) algorithm, where the single function to be approximated by the elements from the dictionary is replaced by the unit ball of the space V_n.

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