An algebraic approach to polynomial reproduction of Hermite subdivision schemes

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In this talk we discuss the capability of a Hermite subdivision scheme to reproduce polynomials in the sense that, for initial data sampled from a polynomial function, the scheme yields the same polynomial and its derivatives in the limit. The polynomial reproduction guarantees that the scheme satisfies the so-called spectral condition that allows factorization of the subdivision operator which in the end leads to convergence results [1]. Moreover, it is well-know that the polynomial reproduction is strictly connected to the approximation order of the scheme, even in the Hermite case [2].

Our study, of purely algebraic nature, provides algebraic conditions on the subdivision symbol and its derivatives for computing the exact degree of polynomial reproduction and also for determining the associated correct parametrization. In this respect, it generalizes the work done in [3] where the polynomial reproduction of a *scalar* subdivision scheme is considered in full generality.

The case we study here, is the case of a Hermite scheme dealing with function values and first derivatives or function values, first and second derivatives. Essentially, these are the known existing Hermite schemes. However, our algebraic approach can be extended to a general situation where function values and derivatives of order higher than 2 are considered.

Several examples of application of the proposed algebraic conditions are given in both the primal and the dual situation.

Joint work with: Svenja Hüning, Institut f. Geometrie, TU Graz, Austria.

References

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