Semi-regular Subdivision:
Smoothness Analysis via Wavelet Frames

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The study of Hölder and Sobolev regularity of univariate regular subdivision has been investigated deeply in the last 25 years using Fourier analysis, wavelet analysis, difference schemes and joint spectral radius techniques. However the shift-invariance of the underlying regular setting is crucial for these techniques. In [1], a general laborious framework for determining the Hölder regularity of irregular subdivision via difference schemes has been proposed. We propose an efficient method based on Dubuc-Deslauriers wavelet tight frame decomposition techniques for approximating the Hölder and Sobolev regularity of univariate semi-regular subdivision schemes. In the semi-regular setting, after a finite number of subdivision steps, the subdivision process is locally a stationary subdivision on an initial mesh of the type $-h_\ell\mathbb{N} \cup \{0\} \cup h_r\mathbb{N}$, $h_\ell, h_r > 0$, which implies that some of the basic limit functions are not integer shifts of other ones.
Univariate frames are special function families $\{\psi_j\}_{j\in J} \subset L_2(\mathbb{R})$ (J is a countable index set) that are used for decomposition and analysis of functions in $L_2(\mathbb{R})$. In the shift-invariant setting, if the frame $\{\psi_j\}_{j\in J} \subset C^s(\mathbb{R})$, $s \in \mathbb{N}$, has s vanishing moments, i.e.

$$(x^k, \psi_j)_{L_2} = 0 \quad \text{for all} \quad k \in \{0, \ldots, s-1\},$$

then the decay rate of the frame coefficients $(\cdot, \psi_j)_{L_2}$, $j \in J$, characterizes Hölder spaces $C^r(\mathbb{R})$ and Sobolev spaces $H^r(\mathbb{R})$ for $r \in (0, s)$.
Our construction of wavelet tight frames, based on the semi-regular Dubuc-Deslauriers $2n$-point interpolatory subdivision schemes, $n \in \mathbb{N}$, amounts to a modification of the Unitary Extension Principle conformed with the characterization of wavelet tight frames in [2]. The interpolation and polynomial reproduction properties of Dubuc-Deslauriers schemes ensure n vanishing moments for the corresponding framelets. We also extend the wavelet based characterization of function spaces in [3] and [4] to the semi-regular case. Furthermore we present proper tools for computing the frame coefficients in the semi-regular setting, generalizing the ideas in [5], [6], [7].

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