Sparse high-dimensional FFT based on rank-1 lattice sampling and Prony's method

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In this talk, we suggest approximate algorithms for the reconstruction of sparse high-dimensional trigonometric polynomials, where the support in frequency domain is unknown. Based on ideas of constructing rank-1 lattices component-by-component, we adaptively construct the index set of frequencies belonging to the non-zero Fourier coefficients in a dimension incremental way. When we restrict the search space in frequency domain to a full grid $[-N, N]^d \cap \mathbb{Z}^d$ of refinement $N \in \mathbb{N}$ and assume that the cardinality of the support of the trigonometric polynomial in frequency domain is bounded by the sparsity $s \in \mathbb{N}$, our method requires $\mathcal{O}(d\, s^2 N)$ samples and $\mathcal{O}(d\, s^3 + d\, s^2 N \log(s\, N))$ arithmetic operations in the case $\sqrt{N} \lesssim s \lesssim N^d$. Moreover, we discuss possibilities to reduce the number of samples and arithmetic operations by applying Prony's method. The number of samples is reduced to $\mathcal{O}(d\, s + d\, N)$ and the number of arithmetic operations is $\mathcal{O}(d\, s^3)$ in this case. Various numerical examples demonstrate the efficiency of the suggested method.

Joint work with: Lutz Kämmerer, Toni Volkmer

References

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