Blending spline transformation of curves and surfaces

Arne Lakså
University of Tromsø, The Arctic University of Norway
arne.laksa@uit.no

In 2005, Expo-rational B-splines were introduced, a blend-spline type of order 2, equivalent to ordinary linear B splines, but where the control points were replaced by local curves or surfaces, [1]. In the following years, the theory was extended from curves and tensor product surfaces to include triangular surfaces, surfaces based on radial basic functions, surfaces on irregular grid and volumes. With regard to expo rational basic functions, the theory was expanded a few years later with Beta function B-splines, a polynomial blending function. In 2015, a complete generalization was introduced, where B-functions and their properties were introduced, [2].

Here we will use the framework of the blending spline, but now on any curve or tensor product surface in order to edit (change shape) on these curves/surfaces. The local curves and surfaces are replaced by interpolation points on a given curve or surface. In the curve case, the procedure is as follows:

- Given a curve $c(t)$ (we call this the original curve).
- We then insert a knot vector $\{t_i\}_{i=0}^{n+1}$ to the curve. If the curve is open, we set the two first knots equal to the start parameter value of the curve, and the two last knots equal to the end parameter value. The knot vector divide the domain into $n-1$ intervals.
- At each internal knot, $\{t_i\}_{i=1}^{n}$, we now have a point $c(t_i)$ on the curve.
- To each of these n points we connect an homogenous $k \times k$ matrix, when the curve is embedded in $\mathbf{R}^{k-1}$.

To compute the edited curve $\widehat{c}(t)$, we first find the index j such that $t_j \leq t < t_{j+1}$,
the expression is:

$$\widehat{c}(t) = [M_i + B \circ w_i(t)(M_{i+1} - M_i)]\ c(t)$$

where B is a B-function, M_i is the homogeneous matrix at the knot t_i, the index i is determined by $t_j \leq t < t_{j+1}$ and

$$w_i(t) = \frac{t - t_i}{t_{i+1} - t_i}.$$

A B-functions is a monotone permutation function $B : I \to I, \quad I = [0,1] \subset \mathbf{R}$, and where a given number of subsequent derivatives are zero at start and end.

The result is that all parameterized curves and surfaces can be edited, the shape can be changed, by inserting interpolating points that can be translated, scaled and rotated (affine maps). The concept of this will be discussed further.

References

[1] A. Lakså, B. Bang, L.T. Dechevsky. Exploring expo-rational B-splines for curves and surfaces. *In Mathematical methods for curves and surfaces, edited by M. Dæhlen, K. Mørken, and L. Schumaker, Nashboro Press*, pp. 253–262, 2005.

[2] A. Lakså. Non polynomial B-splines. *in 41th International Conference - Applications of Mathematics in Engineering and Economics - AMEE13, American Institute of Physics (AIP)*, 1690 pp. 030001, 2015.