NURBS and Iterated Functions Systems

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Iterated Function Systems [1, 2] are a standard tool to generate fractal shapes. Extensions have been proposed to have a more accurate control of the iterative process: recurrent IFS[3], LRIFS[4, 5], and CIFS[6]. CIFS, based on automata, can represent most of standard surfaces like subdivision surfaces [7, 8], Bézier surfaces, Spline surfaces [9] as well as curves, surfaces and volumes. It can also be used to design lighter objects by producing lacunar structures or to build arborescent structures supporting given surfaces as in Figure 1.

This work focuses on a CIFS approach of Non-Uniform B-Splines (NURBS) which are the main used representation in CAD Systems. By analyzing the recursive generating process of basis functions we show that the computation of NURBS is stationary. This implies that NURBS can be represented as a finite automaton of a CIFS. The associate subdivision matrices are directly deduced from blossoming formulation [10] and expressed as a function of the initial nodal vector. Automatons for quadratic and cubic cases are presented in Figure 2 and a generalized method of construction for any degree based on blossoming-form is explained. Non-uniform surfaces can also be generated by deducing the associate automaton which is a "tensor product like" of curves automatons whose surface transition matrices are also products of curve transition matrices.

Figure 1: Arborescent structure supporting a Bézier surface.

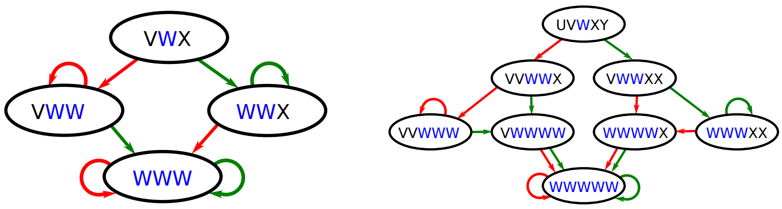

Figure 2: CIFS automaton of quadratic (left) and cubic (right) NURBS curves centered on "W" internode. Left transformations are represented by a red arrow and right transformation by a green one. Whatever the automaton and addresses, this always ends in a, potentially uniform, stationary case.

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