Computing the joint spectral radius of large matrices: practical issues

Thomas Mejstrik
University of Vienna
tommsch@gmx.at

The problem of computation of the joint spectral radius (JSR) of several matrices plays an exceptional role in the theory of refinable surfaces, subdivision schemes, wavelets, etc. In particular, the JSR characterizes the smoothness of refinable curves and surfaces and the convergence of subdivision schemes. However, the problem of the JSRs computation is notoriously hard. We apply to this problem the invariant polytope algorithm [4], which finds the value of the JSR for most matrix families even in relatively high dimensions. That algorithm attempts to construct an extremal norm to find the exact value of the JSR. We propose several modifications of the original implementation [4, 5], including parallelization, natural selection of new vertices, automated adding of extra vertices and an adaption to higher dimensional problems. Furthermore, we present a new method of finding the spectral maximizing product based on Gripenberg's branch-and-bound algorithm [3]. We show that these modifications do not influence the advantages of the original algorithm, illustrate the efficiency of each modification by various numerical examples and compare the modified algorithm to other known algorithms [1, 2, 3, 6].

Joint work with: V. Yu. Protasov, University of L'Aquila (Italy) and Moscow State University (Russia)

References

[1] A. Ahmadi, R. Jungers, P. A. Parrilo, M. Roozbehani, *Joint spectral radius and path-complete graph Lyapunov functions*, SIAM J. Control and Optimization, 52-1 (2014), 687-717, 2014.

[2] A. Cicone, N. Guglielmi, R. Jungers, *Lifted polytope methods for the computation of joint spectral characteristics*, preprint, 2011. SIAM J. Matrix Anal. A, 35 (2014), 391–410

[3] G. Gripenberg, *Computing the joint spectral radius*, Linear Algebra Appl., 234 (1996), 43–60.

[4] N. Guglielmi, V. Yu. Protasov, *Exact computation of joint spectral characteristics of matrices*, Found. Comput. Math., 13 (2013), 37–97.

[5] N. Guglielmi, V. Yu. Protasov, *Invariant polytopes of linear operators with applications to regularity of wavelets and of subdivisions*, SIAM J. Matrix Anal., 37 (2016), no 1, 18–52.

[6] P. A. Parrilo, A. Jadbabaie, *Approximation of the joint spectral radius using sum of squares*, Linear Algebra Appl., 428-10 (2008), 2385–2402.

[7] G. C. Rota, G. Strang, *A note on the joint spectral radius*, Kon. Nederl. Acad. Wet. Proc., 63 (1960), 379–381.