Vector versions of Prony's algorithm

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Given the scalar sequence $\{f_m\}_{m=0}^{\infty}$ that satisfies

$$f_m = \sum_{j=1}^{k} a_j \zeta_j^m, \quad m = 0, 1, \ldots,$$

where $a_j, \zeta_j \in \mathbb{C}$, $j = 1, \ldots, k$, are independent of m, and ζ_j are distinct, the algorithm of Prony concerns the determination of the a_j and the ζ_j from the f_m. This algorithm is also related to Padé approximants from the infinite power series $\sum_{i=0}^{\infty} f_i z^i$.

In this talk, we discuss ways of extending Prony's algorithm to sequences of vectors $\{\boldsymbol{f}_m\}_{m=0}^{\infty}$ in $\mathbb{C}^N$ that satisfy

$$\boldsymbol{f}_m = \sum_{j=1}^{k} \boldsymbol{a}_j \zeta_j^m, \quad m = 0, 1, \ldots,$$

where $\boldsymbol{a}_j \in \mathbb{C}^N$ and $\zeta_j \in \mathbb{C}$, $j = 1, \ldots, k$. Two distinct problems arise depending on whether the vectors $\boldsymbol{a}_j$ are linearly independent or not. We consider different approaches that enable us to determine the $\boldsymbol{a}_j$ and ζ_j for these two problems, and develop suitable methods. We concentrate especially on extensions that take into account the possibility of the components of the $\boldsymbol{a}_j$ being coupled. One of the applications concern the determination of a number of the pairs $(\zeta_j, \boldsymbol{a}_j)$ for which $|\zeta_j|$ are largest. Finally, we consider the more general case in which

$$\boldsymbol{f}_m = \sum_{j=1}^{k} \boldsymbol{p}_j(m) \zeta_j^m, \quad m = 0, 1, \ldots,$$

where $\boldsymbol{p}_j(m) \in \mathbb{C}^N$ are some (vector-valued) polynomials in m, and $\zeta_j \in \mathbb{C}$, $j = 1, \ldots, k$.

Finally, the methods suggested here can be extended to vector sequences in infinite dimensional spaces in a straightforward manner.