Smoothness of refinable surfaces, Minkowski dimension, and synchronizing automata

Vladimir Protasov
University of L'Aquila (Italy), Moscow State University (Russia)
v-protassov@yandex.ru

Multivariate refinable surfaces are constructed as limit functions of subdivision algorithms. They satisfy refinement equations on $\mathbf{R}^d$ with an integer dilation expansive $d \times d$ matrix M and certain set of $m = |\det M|$ "digits" from the corresponding quotient sets. Computation of their smoothness is a hard problem especially for anisotropic matrices M.

A formula for the Hölder exponent in case of general dilation matrices was proved in [1]. We consider a special case when a refinable function is an indicator function of a compact set G in $\mathbf{R}^d$. In this case G is a self-similar attractor. We show that the L_2 Hölder exponent can be expressed by the boundary Minkowski dimension of G. Moreover, the same value has an interpretation in terms of the problem of synchronizing automata. A finite automata is determined by a directed multigraph with N vertices (states) and with all edges (transfers) coloured with m colours so that each vertex has precisely one outgoing edge of each colour. The automata is synchronizing if there exists a finite sequence of colours such that all paths following that sequence terminate at the same vertex independently of the starting vertex. The problem of synchronizing automata has been studied in great detail (see [2] for a survey). It turns out that each Haar function can be naturally associated with a finite automata and the Hölder exponent is related to the length of the synchronizing sequence. We introduce a concept of synchronizing rate and show that it is actually equal to the Hölder exponent of the corresponding Haar function. Applying this result we prove that the Hölder exponent can be found within finite time by a combinatorial algorithm.

References

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