A Deterministic DCT Algorithm for Vectors with Short Support

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There are many well-known fast algorithms for the discrete Fourier transform (DFT) of sparse input signals or vectors, both deterministic and randomized ones. The fastest of the deterministic algorithms achieve runtimes of $\mathcal{O}\left(m^2 \log^{\mathcal{O}(1)} N\right)$ for a general m-sparse input signal of length N. If it is even known that the input signal has a short support of length m, the runtime can be reduced to $\mathcal{O}\left(m \log N\right)$.

For the closely related discrete cosine transform of type II (DCT-II), given by

$$\mathbf{x}^{\widehat{\mathrm{II}}} := \sqrt{\frac{2}{N}} \left(\varepsilon_N(j) \cos\left(\frac{j(2k+1)\pi}{2N} \right) \right)_{j,\,k=0}^{N-1} \mathbf{x}, \quad \mathbf{x} \in \mathbb{R}^N,$$

where $\varepsilon_N(0) = 1/\sqrt{N}$ and $\varepsilon_N(j) = 1$ for $j \neq 0$, no such algorithms are known so far. In this talk we present a new fast and deterministic DCT-II algorithm that reconstructs the input vector $\mathbf{x} \in \mathbb{R}^N_{\geq 0}$, $N = 2^J$, with short support of length m from $\mathbf{x}^{\widehat{\mathrm{II}}}$ by combining ideas from two sparse FFT algorithms for vectors with short support by Plonka and Wannenwetsch in [1] and [2]. The resulting algorithm has a runtime of $\mathcal{O}(m \log m \log N)$ and requires $\mathcal{O}(m \log N)$ samples of $\mathbf{x}^{\widehat{\mathrm{II}}}$.

Joint work with: Gerlind Plonka.

References

[1] G. Plonka and K. Wannenwetsch. A deterministic sparse FFT algorithm for vectors with small support. *Numerical Algorithms*, 71(4):889-905, 2016.

[2] G. Plonka and K. Wannenwetsch. A sparse fast Fourier algorithm for real non-negative vectors. *Journal of Computational and Applied Mathematics*, 321:532 539, 2017.