A convex approach to super-resolution and regularization of lines in images

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Geometric structures such as contours must be able to be detected to locate objects in 2-D images. One of the competitive issues currently is that of super-resolution, consisting in restoring lost details from low resolution images. Although the super-resolution of source points has been intensively exploited in recent years [1], the super-resolution of curves is still in its infancy. Applications concern both HD television and blurred images due to diffraction in microscopy or astrophysics. Our motivation stems from the frequent presence in biomedical images of elongated structures like filaments, neurons, veins, deteriorated during the acquisition process.

We present a new convex formulation for the problem of recovering lines in degraded images [2, 3]. The underlying model is that of a superposition of diffracted lines (convoluted by a blur kernel) (Fig. 1a,1b) with presence of noise (Fig. 1c,1d), whose position and intensity parameters must be recovered with sub-pixel precision (Fig. 1e, 1f). This is made possible by a regularization of the inverse problem favoring some notion of parcimony. An elegant and unifying formulation, which yields convex problems, is based on the general framework of *atomic norm* minimization [4], enforcing the solution to be expressed as a sparse positive combination of elements, called *atoms*, belonging to an infinite dictionary indexed by continuously varying parameters.

In this work, we consider the setting where the atoms are *lines*. Expressed in the Fourier domain, these atoms can be characterized with respect to their rows and columns, and the problem can be reduced to a dictionary of 1-D complex exponential samples, indexed by their frequency and phase, and the atomic norm can be computed via semidefinite programming. This formulation enables us to derive a convex optimization problem under constraints, solved by mean of a splitting primal-dual algorithm. Then, performing a Prony-like method onto the solution of the algorithm allows us to extract the parameters of the lines. This approach provides a very high accuracy for the line estimation, where the Hough transform fails, due to its discrete nature. This gives a proof of concept for a revised Hough transform devolved to a continuous curves detection.

Joint work with: Laurent Condat, Marianne Clausel, Valérie Perrier.

References

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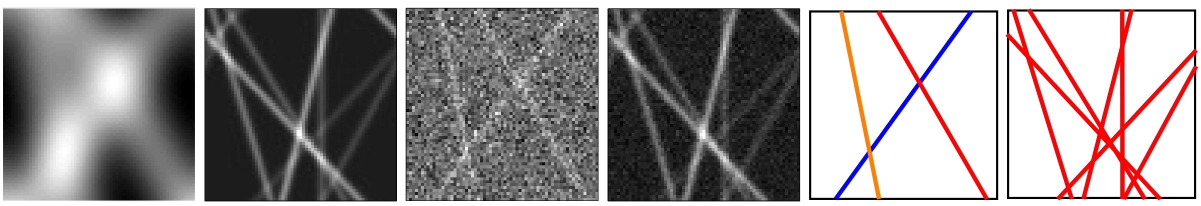

(a) (b) (c) (d) (e) (f)

Figure 1: (a)-(b)-(d)-(e) Images of blurred and/or noisy lines, (c)-(f) Line detection with subpixel accuracy.